CYCLOTOMIC HEPTASECTION FOR THE PRIME 43.

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The problem of cyclotomic section has engaged the attention of many eminent mathematicians and solutions have been obtained by them for particular cases. The problem of the trisection and quartisection was completely solved by Cayley in a paper in which he also discussed the quinquisection but did not complete the solution. He once again took up the same problem in the proceedings of the London Mathematical Society in 1881 but was not able to complete the solution.

The problem of quinquisection was first solved by Rogers.† The same problem has very recently been solved by Burnside.‡ Towards the end of his paper he refers to the case of heptasection and says "I have carried the case $q = 7$ so far as to assure myself that it is not quite parallel with that of $q = 5$; a set of three simultaneous Diophantine equations occur, but they are not sufficient to ensure that the equations expressing the product of A's form a consistent multiplication table." In view of this statement it is believed that the problem of heptasection for the prime 43 has not been previously considered.

The object of this paper is to consider the problem of heptasection for the prime 43. All the details of calculation have been suppressed in order to save space, and only the final result is given.

Let a be an imaginary root of $x^{43} - 1 = 0$, and let us divide all the imaginary roots into 7 groups according to the following scheme:

$$\begin{aligned}
A &= a + a^{42} + a^{37} + a^{6} + a^{36} + a^{7},\\
B &= a^{3} + a^{40} + a^{25} + a^{18} + a^{22} + a^{21},\\
C &= a^{9} + a^{34} + a^{32} + a^{11} + a^{23} + a^{20},\\
D &= a^{27} + a^{16} + a^{10} + a^{33} + a^{26} + a^{17},\\
E &= a^{38} + a^{5} + a^{30} + a^{13} + a^{35} + a^{8},\\
F &= a^{28} + a^{15} + a^{4} + a^{39} + a^{19} + a^{24},\\
G &= a^{41} + a^{2} + a^{12} + a^{31} + a^{14} + a^{29}.
\end{aligned}$$

Calculating the elementary symmetric functions of these expressions we get:

$$\sum A = -1, \quad \sum AB = -18, \quad \sum ABC = 35, \quad \sum ABCD = 38,$$
$$\sum ABCDE = -104, \quad \sum ABCDEF = 7, \quad ABCDEFG = 49.$$

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† Lond. Math. Soc. Proc., vol. 32 (1900–01), pp. 199–207.

‡ Lond. Math. Soc. Proc., vol. (2) 14, (1915), pp. 251–259.

The equation whose roots are A, B, C, D, E, F, G is therefore

$$\eta^7 + \eta^6 - 18\eta^5 - 35\eta^4 + 38\eta^3 + 104\eta^2 + 7\eta - 49 = 0.$$

Every root of this equation may be expressed as a rational integral function of any one assigned root; it is therefore an Abelian equation and can be solved by radicals.

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